7 TECHNICAL DETAILS OF SKETCH PLANE WIDGET

Below are helpful equations and illustrations that describe how 2D touches are mapped to 3D manipulations for translating and rotating the sketch plane widget (π) with respect to the selected axis (π_a), bezel (π_b), or center (π_c) with 1 or 2 fingers.

7.1 Rotation About an Axis (Fig. 6b)

A sketch plane widget π is defined in terms of a position $\mathbf{X}$, normal $\mathbf{n}$, and two axial directions $\mathbf{a}_1$ and $\mathbf{a}_2$. Its movement can be expressed as:

$$\pi : \{\mathbf{X}, \mathbf{n}, \mathbf{a}_1, \mathbf{a}_2\} \rightarrow \pi' : \{\mathbf{X}', \mathbf{n}', \mathbf{a}_1', \mathbf{a}_2'\}$$

When an axis π_a with the direction $\mathbf{a}_1$ is selected and a touch is made at $\mathbf{T}_1$, the touch point is projected as $\mathbf{P}_1$ by the intersection of a line $\mathrm{l}(\mathbf{E}, \mathbf{T}_1)$ that connects the eye $\mathbf{E}$ to $\mathbf{T}_1$ with a plane $\mathrm{p}(\mathbf{X}, \mathbf{n})$ that contains $\mathbf{X}$ and has the normal $\mathbf{n}$:

$$\mathbf{P}_1 = \mathrm{l}(\mathbf{E}, \mathbf{T}_1) \cap \mathrm{p}(\mathbf{X}, \mathbf{n})$$

Then, a second plane $\mathrm{p}(\mathbf{C}, \mathbf{a}_1)$ is defined, where the position $\mathbf{C}$ it contains is calculated as:

$$\mathbf{C} = \mathbf{X} + ((\mathbf{P}_1 - \mathbf{X}) \cdot \mathbf{a}_1)\mathbf{a}_1$$

When the touch is dragged to $\mathbf{T}_1'$, it is projected onto the second plane $\mathrm{p}(\mathbf{C}, \mathbf{a}_1)$ as $\mathbf{P}_1'$:

$$\mathbf{P}_1' = \mathrm{l}(\mathbf{E}, \mathbf{T}_1') \cap \mathrm{p}(\mathbf{C}, \mathbf{a}_1)$$

Finally, the sketch plane widget π is moved to π' so that:

$$\mathbf{X}' = \mathbf{X}$$
$$\mathbf{a}_1' = \mathbf{a}_1$$
$$\mathbf{a}_2' = \frac{\mathbf{P}_1' - \mathbf{C}}{\|\mathbf{P}_1' - \mathbf{C}\|}$$
$$\mathbf{n}' = \mathbf{a}_1' \times \mathbf{a}_2'$$

7.2 Translation Along an Axis (Fig. 6c)

A sketch plane widget π is defined in terms of a position $\mathbf{X}$, normal $\mathbf{n}$, and two axial directions $\mathbf{a}_1$ and $\mathbf{a}_2$. Its movement can be expressed as:

$$\pi : \{\mathbf{X}, \mathbf{n}, \mathbf{a}_1, \mathbf{a}_2\} \rightarrow \pi' : \{\mathbf{X}', \mathbf{n}', \mathbf{a}_1', \mathbf{a}_2'\}$$

When an axis π_a with the direction $\mathbf{a}_1$ is selected and two touches are made at $\mathbf{T}_1$ and $\mathbf{T}_2$, the touch points are projected as $\mathbf{P}_1$ and $\mathbf{P}_2$ by the intersections of lines $\mathrm{l}(\mathbf{E}, \mathbf{T}_1)$ and $\mathrm{l}(\mathbf{E}, \mathbf{T}_2)$ that connect the eye $\mathbf{E}$ to $\mathbf{T}_1$ and $\mathbf{T}_2$ with a plane $\mathrm{p}(\mathbf{X}, \mathbf{n})$ that contains $\mathbf{X}$ and has the normal $\mathbf{n}$:

$$\mathbf{P}_1 = \mathrm{l}(\mathbf{E}, \mathbf{T}_1) \cap \mathrm{p}(\mathbf{X}, \mathbf{n})$$
$$\mathbf{P}_2 = \mathrm{l}(\mathbf{E}, \mathbf{T}_2) \cap \mathrm{p}(\mathbf{X}, \mathbf{n})$$

The midpoint between the two points $\mathbf{P}_1$ and $\mathbf{P}_2$ is calculated as $\mathbf{P}_\mathrm{m}$:

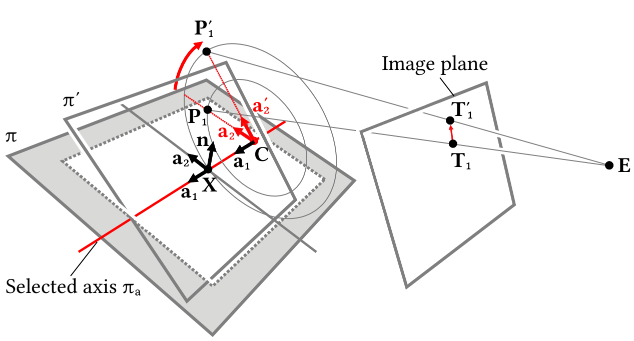
$$\mathbf{P}_\mathrm{m} = \frac{\mathbf{P}_1 + \mathbf{P}_2}{2}$$

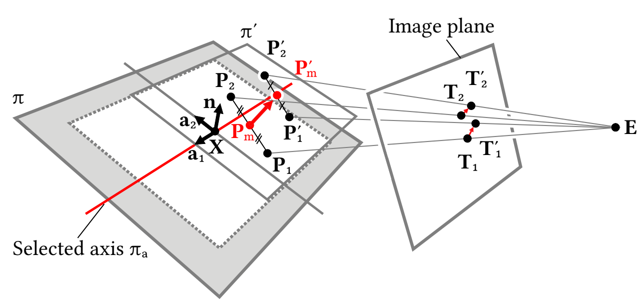
When the touches are dragged to $\mathbf{T}_1'$ and $\mathbf{T}_2'$, their projections on the same plane $\mathrm{p}(\mathbf{X}, \mathbf{n})$ and the midpoint between those projections are calculated as $\mathbf{P}_1'$, $\mathbf{P}_1'$, and $\mathbf{P}_\mathrm{m}'$:

$$\mathbf{P}_1' = \mathrm{l}(\mathbf{E}, \mathbf{T}_1') \cap \mathrm{p}(\mathbf{X}, \mathbf{n})$$
$$\mathbf{P}_2' = \mathrm{l}(\mathbf{E}, \mathbf{T}_2') \cap \mathrm{p}(\mathbf{X}, \mathbf{n})$$
$$\mathbf{P}_\mathrm{m}' = \frac{\mathbf{P}_1' + \mathbf{P}_2'}{2}$$

Finally, the sketch plane widget π is moved to π' so that:

$$\mathbf{X}' = \mathbf{X} + ((\mathbf{P}_\mathrm{m}' - \mathbf{P}_\mathrm{m}) \cdot \mathbf{a}_1)\mathbf{a}_1$$
$$\mathbf{a}_1' = \mathbf{a}_1$$
$$\mathbf{a}_2' = \mathbf{a}_2$$
$$\mathbf{n}' = \mathbf{n}$$





7.3 Translation on the Plane (Fig. 6e)

A sketch plane widget π is defined in terms of a position $\mathbf{X}$, normal $\mathbf{n}$, and two axial directions $\mathbf{a}_1$ and $\mathbf{a}_2$. Its movement can be expressed as:

$$\pi : \{\mathbf{X}, \mathbf{n}, \mathbf{a}_1, \mathbf{a}_2\} \rightarrow \pi' : \{\mathbf{X}', \mathbf{n}', \mathbf{a}_1', \mathbf{a}_2'\}$$

When the bezel π_b is selected and a touch is made at $\mathbf{T}_1$, the touch point is projected as $\mathbf{P}_1$ by the intersection of a line $\mathrm{l}(\mathbf{E}, \mathbf{T}_1)$ that connects the eye $\mathbf{E}$ to $\mathbf{T}_1$ with a plane $\mathrm{p}(\mathbf{X}, \mathbf{n})$ that contains $\mathbf{X}$ and has the normal $\mathbf{n}$:

$$\mathbf{P}_1 = \mathrm{l}(\mathbf{E}, \mathbf{T}_1) \cap \mathrm{p}(\mathbf{X}, \mathbf{n})$$

When the touch is dragged to $\mathbf{T}_1'$, it is projected on the same plane $\mathrm{p}(\mathbf{X}, \mathbf{n})$ as $\mathbf{P}_1'$:

$$\mathbf{P}_1' = \mathrm{l}(\mathbf{E}, \mathbf{T}_1') \cap \mathrm{p}(\mathbf{X}, \mathbf{n})$$

Finally, the sketch plane widget π is moved to π' so that:

$$\begin{aligned} \mathbf{X}' &= \mathbf{X} + (\mathbf{P}_1' - \mathbf{P}_1) \\ \mathbf{a}_1' &= \mathbf{a}_1 \\ \mathbf{a}_2' &= \mathbf{a}_2 \\ \mathbf{n}' &= \mathbf{n} \end{aligned}$$

7.4 Translation and Rotation on the Plane (Fig. 6f)

A sketch plane widget π is defined in terms of a position $\mathbf{X}$, normal $\mathbf{n}$, and two axial directions $\mathbf{a}_1$ and $\mathbf{a}_2$. Its movement can be expressed as:

$$\pi : \{\mathbf{X}, \mathbf{n}, \mathbf{a}_1, \mathbf{a}_2\} \rightarrow \pi' : \{\mathbf{X}', \mathbf{n}', \mathbf{a}_1', \mathbf{a}_2'\}$$

When the bezel π_b is selected and two touches are made at $\mathbf{T}_1$ and $\mathbf{T}_2$, the touch points are projected as $\mathbf{P}_1$ and $\mathbf{P}_2$ by the intersections of lines $\mathrm{l}(\mathbf{E}, \mathbf{T}_1)$ and $\mathrm{l}(\mathbf{E}, \mathbf{T}_2)$ that connect the eye $\mathbf{E}$ to $\mathbf{T}_1$ and $\mathbf{T}_2$ with a plane $\mathrm{p}(\mathbf{X}, \mathbf{n})$ that contains $\mathbf{X}$ and has the normal $\mathbf{n}$:

$$\begin{aligned} \mathbf{P}_1 &= \mathrm{l}(\mathbf{E}, \mathbf{T}_1) \cap \mathrm{p}(\mathbf{X}, \mathbf{n}) \\ \mathbf{P}_2 &= \mathrm{l}(\mathbf{E}, \mathbf{T}_2) \cap \mathrm{p}(\mathbf{X}, \mathbf{n}) \end{aligned}$$

The midpoint between the two points $\mathbf{P}_1$ and $\mathbf{P}_2$ is calculated as $\mathbf{P}_\mathrm{m}$:

$$\mathbf{P}_\mathrm{m} = \frac{\mathbf{P}_1 + \mathbf{P}_2}{2}$$

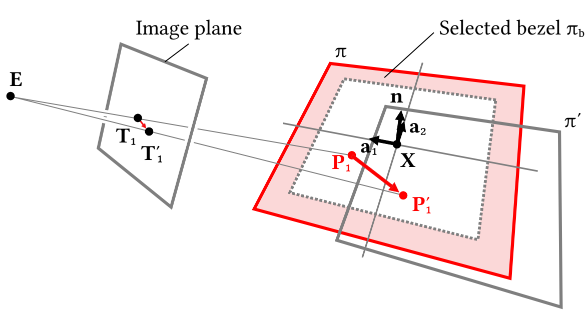
When the touches are dragged to $\mathbf{T}_1'$ and $\mathbf{T}_2'$, their projections on the same plane $\mathrm{p}(\mathbf{X}, \mathbf{n})$ and the midpoint between those projections are calculated as $\mathbf{P}_1'$, $\mathbf{P}_2'$, and $\mathbf{P}_\mathrm{m}'$:

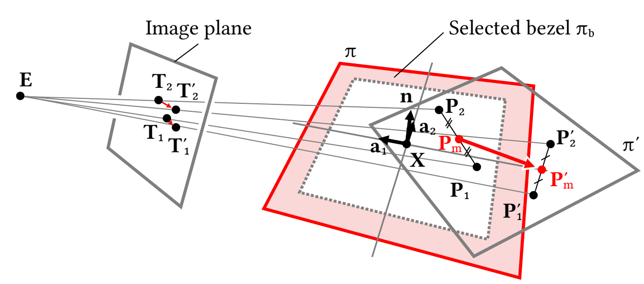
$$\begin{aligned} \mathbf{P}_1' &= \mathrm{l}(\mathbf{E}, \mathbf{T}_1') \cap \mathrm{p}(\mathbf{X}, \mathbf{n}) \\ \mathbf{P}_2' &= \mathrm{l}(\mathbf{E}, \mathbf{T}_2') \cap \mathrm{p}(\mathbf{X}, \mathbf{n}) \\ \mathbf{P}_\mathrm{m}' &= \frac{\mathbf{P}_1' + \mathbf{P}_2'}{2} \end{aligned}$$

Then, a quaternion $\mathbf{q}$ that expresses the rotation of unit vectors from $\dfrac{\mathbf{P}_2 - \mathbf{P}_1}{\|\mathbf{P}_2 - \mathbf{P}_1\|}$ to $\dfrac{\mathbf{P}_2' - \mathbf{P}_1'}{\|\mathbf{P}_2' - \mathbf{P}_1'\|}$ is calculated.

Finally, the sketch plane widget π is moved to π' so that:

$$\begin{aligned} \mathbf{X}' &= \mathbf{X} + (\mathbf{P}_\mathrm{m}' - \mathbf{P}_\mathrm{m}) \\ \mathbf{a}_1' &= \mathbf{q}\mathbf{a}_1\mathbf{q}^{-1} \\ \mathbf{a}_2' &= \mathbf{q}\mathbf{a}_2\mathbf{q}^{-1} \\ \mathbf{n}' &= \mathbf{n} \end{aligned}$$





7.5 Orbit About the Center (Fig. 6h)

A sketch plane widget π is defined in terms of a position $\mathbf{X}$, normal $\mathbf{n}$, and two axial directions $\mathbf{a}_1$ and $\mathbf{a}_2$. Its movement can be expressed as:

$$\pi : \{\mathbf{X}, \mathbf{n}, \mathbf{a}_1, \mathbf{a}_2\} \rightarrow \pi' : \{\mathbf{X}', \mathbf{n}', \mathbf{a}_1', \mathbf{a}_2'\}$$

When the center π_c is selected and a touch is made at $\mathbf{T}_1$, the touch point is projected as $\mathbf{P}_1$ by the intersection of a line $\mathrm{l}(\mathbf{E}, \mathbf{T}_1)$ that connects the eye $\mathbf{E}$ to $\mathbf{T}_1$ with a plane $\mathrm{p}(\mathbf{X}, \mathbf{n})$ that contains $\mathbf{X}$ and has the normal $\mathbf{n}$:

$$\mathbf{P}_1 = \mathrm{l}(\mathbf{E}, \mathbf{T}_1) \cap \mathrm{p}(\mathbf{X}, \mathbf{n})$$

Then, a sphere $\mathrm{s}(\mathbf{X}, \mathrm{r})$ that is located at $\mathbf{X}$ and has a radius of r is defined, where:

$$\mathrm{r} = \|\mathbf{P}_1 - \mathbf{X}\|$$

When the touch is dragged to $\mathbf{T}_1'$, it is projected onto the sphere $\mathrm{s}(\mathbf{X}, \mathrm{r})$ as $\mathbf{P}_1'$:

$$\mathbf{P}_1' = \mathrm{l}(\mathbf{E}, \mathbf{T}_1') \cap \mathrm{s}(\mathbf{X}, \mathrm{r})$$

Then, for the case in which the line $\mathrm{l}(\mathbf{E}, \mathbf{T}_1')$ and sphere $\mathrm{s}(\mathbf{X}, \mathrm{r})$ intersect, a quaternion $\mathbf{q}_o$ that expresses the rotation of unit vectors from $\dfrac{\mathbf{P}_1 - \mathbf{X}}{\|\mathbf{P}_1 - \mathbf{X}\|}$ to $\dfrac{\mathbf{P}_1' - \mathbf{X}}{\|\mathbf{P}_1' - \mathbf{X}\|}$ is calculated.

Finally, the sketch plane widget π is moved to π' so that:

$$\begin{aligned} \mathbf{X}' &= \mathbf{X} \\ \mathbf{a}_1' &= \mathbf{q}_o \mathbf{a}_1 \mathbf{q}_o^{-1} \\ \mathbf{a}_2' &= \mathbf{q}_o \mathbf{a}_2 \mathbf{q}_o^{-1} \\ \mathbf{n}' &= \mathbf{a}_1' \times \mathbf{a}_2' \end{aligned}$$

7.6 Orbit and Spin About the Center (Fig. 6i)

Continuing from the sketch plane widget π' that orbited about the center π_c, additional movement from spinning can be expressed as:

$$\pi' : \{\mathbf{X}', \mathbf{n}', \mathbf{a}_1', \mathbf{a}_2'\} \rightarrow \pi'' : \{\mathbf{X}'', \mathbf{n}'', \mathbf{a}_1'', \mathbf{a}_2''\}$$

When a second touch is made at $\mathbf{T}_2$, it is projected onto the sphere $\mathrm{s}(\mathbf{X}, \mathrm{r})$ as $\mathbf{P}_2$:

$$\mathbf{P}_2 = \mathrm{l}(\mathbf{E}, \mathbf{T}_2) \cap \mathrm{s}(\mathbf{X}, \mathrm{r})$$

Then, for the case in which the line $\mathrm{l}(\mathbf{E}, \mathbf{T}_2)$ and sphere $\mathrm{s}(\mathbf{X}, \mathrm{r})$ intersect, a second plane $\mathrm{p}(\mathbf{C}, \mathbf{n}_c)$ is defined, where the position $\mathbf{C}$ it contains and its normal $\mathbf{n}_c$ are calculated:

$$\begin{aligned} \mathbf{n}_c &= \frac{\mathbf{P}_1' - \mathbf{X}}{\|\mathbf{P}_1' - \mathbf{X}\|} \\ \mathbf{C} &= \mathbf{X} + ((\mathbf{P}_2 - \mathbf{X}) \cdot \mathbf{n}_c)\mathbf{n}_c \end{aligned}$$

When the second touch is dragged to $\mathbf{T}_2'$, it is projected onto the second plane $\mathrm{p}(\mathbf{C}, \mathbf{n}_c)$ as $\mathbf{P}_2'$:

$$\mathbf{P}_2' = \mathrm{l}(\mathbf{E}, \mathbf{T}_2') \cap \mathrm{p}(\mathbf{C}, \mathbf{n}_c)$$

Then, a quaternion $\mathbf{q}_s$ that expresses the rotation of unit vectors from $\dfrac{\mathbf{P}_2 - \mathbf{C}}{\|\mathbf{P}_2 - \mathbf{C}\|}$ to $\dfrac{\mathbf{P}_2' - \mathbf{C}}{\|\mathbf{P}_2' - \mathbf{C}\|}$ is calculated.

Finally, the sketch plane widget π' is moved to π'' so that:

$$\begin{aligned} \mathbf{X}'' &= \mathbf{X}' \\ \mathbf{a}_1'' &= \mathbf{q}_s \mathbf{a}_1' \mathbf{q}_s^{-1} \\ \mathbf{a}_2'' &= \mathbf{q}_s \mathbf{a}_2' \mathbf{q}_s^{-1} \\ \mathbf{n}'' &= \mathbf{a}_1'' \times \mathbf{a}_2'' \end{aligned}$$